Advanced Automation and Control

July 11, 2019

Surname _____ Name _____

Part I - Optimization & Graphs (Prof. D.M. Raimondo)

1. Please solve the following MILP problem using the branch and bound algorithm

$$\begin{array}{ll} \max_{x_1,\delta_1,\delta_2} & -x_1 + 20\delta_1 + \delta_2 \\ & x_1 + \delta_2 + 3\delta_1 \leq 1 \\ & \delta_1, \delta_2 \in \{0,1\} \\ & x_1 \geq 0 \end{array}$$

2. Mr. Grecchi is planning his vacation to the seaside. He found two types of accommodations: an Airbnb and a 3 star hotel. The Airbnb costs 75€/night but does not include breakfast nor a private parking. The price of a breakfast in a local bar is around 6.5€/day. The price of a city parking is 11€/day. The hotel costs 85€/night and includes both the parking and the breakfast. Mr. Grecchi can go to the seaside either by train or by car (in this second case, if he chooses the Airbnb, he will need to pay for the city parking). The cost of the round trip by train is 67€. In case he goes by train, he will take a taxi from the station to his accommodation. The cost of the taxi is 38€ for the round trip. In case he decides to go by car, the cost is of 52€ for the gasoline plus 48€ for the highway. **The objective of Mr. Grecchi is to maximize the number of days he can stay on vacation** taking into account he has a maximum budget of 1000€. Please formulate the problem above as a mixed integer linear program to support the decision-making process of Mr. Grecchi.

3. Consider the following optimisation problem

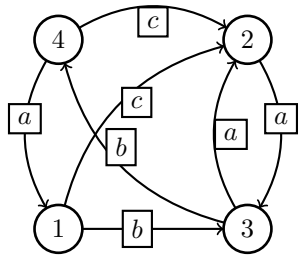
$$\begin{array}{ll} \min_{x_1,x_2} & x_1^2 - x_2^2 + 2x_1 - 4x_2 \\ & x_1^2 + x_2^2 \leq 25 \\ & x_1 + x_2 \leq 10 \end{array}$$

   3.1 Indicate if the cost function is convex (motivate the answer).

   3.2 Depict the feasibility domain of the problem.

   3.3 Indicate if the optimisation problem is convex (motivate the answer).

4. Consider the automaton in the figure ($C = \{a, b, c\}$ is the set of control values and $S = \{1, 2, 3, 4\}$ is the set of state values) with the intermediate cost $g(x, u)$ and the terminal cost $g_3(x)$ given below



$g(x,u)$	a	b	c
1	-	1	1.5
2	1	-	-
3	0.4	3	-
4	2	-	0.5

$$g_3(x) = \begin{cases} 3 & \text{if } x = 1 \\ 0 & \text{if } x = 2 \\ 1 & \text{if } x = 3 \\ 2 & \text{if } x = 4 \end{cases}$$

   2.1 Solve the optimal control problem

$$J(x_0) = \min_{u_0,u_1,u_2} g_3(x_3) + \sum_{k=0}^{2} g(x_k, u_k)$$

   using dynamic programming.

   2.2 Compute the optimal control sequence for $x_0 = 3$ and the corresponding optimal cost value.